LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
Linjär och kombinatorisk optimering
2005-3-11 - 2005-3-21, HEMTENTA

You may use any books and computer programs (e.g. Matlab and Maple), but it is not permitted to get help from other persons. Programs and long calculations can be submitted by e-mail (ufn@maths.lth.se). The data for problems can be received from http://www.maths.lth.se/matematiklth/personal/ufn/kombopt/tentadata.html

Hand in solutions to 6 of 9 problems below. For a passing grade (3), at least 3 problems have to be solved correctly. Credits can be given for partially solved problems. Write your solutions neatly and explain your calculations. Both the content and the format of your solutions, and also how difficult problems you choose, will affect your grade.

The exam has to be handed in to the Student's Expedition in the Mathematics Department at the latest March, 21 at 5.00 p.m. Write your name, section-year (or subject for Ph.D-students), id-number, and phone number or email address on the first page, and write your name on each of the following pages. The result will be announced on the board on the first floor in the Mathematics building on March 31, and those who have given an email address will be notified that way. The oral part of the exam should take place in March-April (depending on your schedule).

Problems can have some parameters $(a, b, c, d \ldots)$. The value of the parameters should be chosen according your personal number (or simply birthday if you have not personal number) abcdeg-****. For example if your personal number is 650327-2384 then $a = 6, b = 5, c = 0, g = 7$ in your problems.

1. Solve the following linear programming problem

$$[-1 \ a \ b \ d\,]\, x \to max$$

with

$$\begin{bmatrix} 6 & -2 & 3 & 5 \\ 2 & 4 & -5 & 6 \\ 2 & -15 & 4 & c \\ 7 & 1 & -4 & 0 \\ 2 & 0 & -6 & 1 \\ 0 & 4 & -3 & 1 \end{bmatrix} x \le \begin{bmatrix} 32 \\ 15 \\ -10 \\ 18 \\ -6 \\ 20 \end{bmatrix}, x \ge 0$$

for example using the two phase method. Are there any feasible solutions? If there is an optimal solution find it. Do the same for the dual problem.

2. Ann and Peter are playing the following game. Each of them independently put in the box $1, 2, 3$ or 4 dollars. Then they open the box. If it will be the prime number of dollars in the box then Peter win and get all the sum. Otherwise it is Ann who get money from the box. What are optimal strategies for each of them and who has more chances to win?

3. A consulting firm has 6 consults. The following table shows the maximum number of days each of this consults can work:

$C1$	$C2$	$C3$	$C4$	$C5$	$C6$
250	200	$150 + 10d$	220	300	$100 + 10b$

The firm has 10 clients, each of them needs 120 days of consulting. The following table shows how much they agree to pay for one day of work of the corresponding consult.

	$C1$	$C2$	$C3$	$C4$	$C5$	$C6$
1	210	200	350	250	100	300
2	200	220	320	230	160	340
3	180	210	300	240	150	330
4	190	220	250	210	130	310
5	210	200	300	190	100	300
6	180	210	330	250	180	320
7	200	220	320	220	170	310
8	190	200	210	200	180	300
9	180	240	320	240	160	290
10	190	250	310	270	140	310

How to organize the consulting to optimize the profit? One consult can work for different clients but not the same day. Several consults can work for the same client.

4. There are $n > 2$ candidates for the post of the head of the department. It is known that for any candidate a there exists a candidate b such that the majority of voters prefer a against b. Prove that there exists three candidates x, y, z such that the majority prefer x against y, the majority prefer y against z and the majority prefer z against x (a Condorocet paradox).

5. Eight problems were proposed to each of 30 students on the exam in the Combinatorial Optimization. To take into account the difficulty level of the problems the professor, when the exam was over, assigned each problem its worth according to the following rule: a problem is worth n points if it is not solved by exactly n contestants. (For example, a problem solved by all students is worth 0 points.) (a) Is it possible that the student having got more points than any other student at the same time has solved less problems than any other student? (b) Is it possible that the student having got less points than any other student at the same time has solved more problems than any other student? (It is supposed that any student can get, for any problem, either zero mark or full mark.)

6. 10 students should choose one of 12 possible scientific advisers for his work. Each applicant have ordered the professors according to his preferences. The following table contains the preference list for every candidate (the most desirable position is written on the first place)

$N1$	6	3	1	10	2	4	7	11	12	9	8	5
$N2$	6	3	2	10	7	11	5	1	9	4	12	8
$N3$	6	10	3	1	11	7	2	8	9	4	5	12
$N4$	6	2	9	3	7	8	11	1	12	10	4	5
$N5$	5	11	3	9	8	7	6	12	10	2	1	4
$N6$	4	1	6	5	11	3	8	9	10	12	2	7
$N7$	6	1	3	2	7	8	12	5	9	10	11	4
$N8$	6	10	4	1	2	11	12	8	9	5	7	3
$N9$	6	7	11	2	12	8	5	10	1	4	3	9
$N10$	3	8	5	11	9	6	1	4	7	2	12	10

After the interview the professors also created a list of their preferences - the most desirable candidate is written on the first place. The list looks like

$P1$	10	4	8	7	5	2	6	3	9	1
$P2$	10	2	9	1	8	3	5	7	4	6
$P3$	2	5	7	6	1	10	3	4	8	9
$P4$	10	7	1	2	8	6	3	5	9	4
$P5$	6	2	7	4	9	1	10	5	3	8
$P6$	7	8	10	1	9	4	3	6	2	5
$P7$	6	10	8	4	7	1	2	3	9	5
$P8$	8	9	6	2	1	4	10	7	3	5
$P9$	10	2	8	6	4	7	9	5	3	1
$P10$	9	10	7	8	2	3	6	1	5	4
$P11$	6	10	8	4	9	7	2	3	1	5
$P12$	6	10	7	4	9	1	2	5	3	8

The head of the department decided that no one professor can get more then one student. But he do not like conflicts, which appear when some student can find a scientific advisor which he prefers to the adviser he was assigned and simultaneously this professor prefer him to the candidate that was assigned to him or has no assigned students at all. Is it possible to make an assignment without any conflicts? Is it possible to do it in different ways?

7. You need to find the most safe route to send your message from the station A to the station H. You can use the intermediate stations B-G. The probability to lose the information sending it between different stations directly is collected in the following table:

$from$	$to\ A$	$to\ B$	$to\ C$	$to\ D$	$to\ E$	$to\ F$	$to\ G$	$to\ H$
A	0	$0.01 * a$	0.15	0.3	0.35	0.45	0.56	0.64
B	0.05	0	0.1	0.18	0.29	0.38	0.46	0.54
C	0.15	$0.02 * b$	0	0.11	0.19	0.28	0.33	0.44
D	0.25	0.18	0.08	0	0.09	0.2	0.23	0.40
E	0.35	0.26	0.2	0.07	0	$0.02 * c$	0.13	0.24
F	0.45	0.38	0.3	0.21	0.09	0	0.07	0.14
G	0.55	0.45	0.4	0.31	0.19	$0.02 * d$	0	0.09
H	0.65	0.55	0.4	0.41	0.29	0.18	0.08	0

8. A thief in the airport wants to open the bag that has 3-digits lock. He can change one digit in 0.5 second and has only 8.5 minutes. The bags opens automatically when the correct digits are chosen. Originally there are digits 000 on the lock and it is closed. He decided to check all the codes one after another in their natural order: $001, 002, 003, \ldots, 999$. Has he time for this? (Note that, for example, to switch from 019 to 020 he changes two digits and needs one second.)

What is the minimal time he needs if he can use another order of codes to reduce the number of digit changes? Give an example of such an order (or describe how to create it).

9. You have a balance and need to choose 5 weights to be able to weigh all objects with the integer weight between 1 and N. What is the maximum value of N if

a)the weights can be placed on the right scale only?

b)the weights can be on both scales?

For example, using weights $1, 2, 5$ you can create all the weights between 1 and 8 in the case b), but cannot create a weight 4 in the case a).

Note that you need to prove that you choice of weights is the best one.