

No books, notes, computational devices, etc. are allowed. Use only paper supplied by the department. Use clear handwriting and give clear careful motivations. All answers should be fully simplified, and in particular they should not contain binomial coefficients, Stirling numbers or factorials. They may, however, contain powers a^b . Fill in the form completely and write your personal identifier on each sheet of paper.

1. How many integer solutions does the equation $x_1 + x_2 + x_3 + x_4 = 12$ have under the conditions
 - a) All x_i are non-negative.
 - b) $x_1 \geq 5$ and $x_i \geq -1$ for $2 \leq i \leq 4$.
 - c) All x_i are non-negative and x_1 is even.
2. How many arrangements of the letters in COOKBOOKS does not contain any of the words COOK, BOOK or SOCK?
3. Let $s_n = \sum_{k=2}^n \binom{k}{2}$ for $n \geq 2$.
 - a) Find a recurrence relation with an initial condition for s_n .
 - b) Solve the recurrence from **a**).
4. Find all solutions to the system of congruencies

$$\begin{cases} 3x + 2 \equiv 1 \pmod{5} \\ 5x + 1 \equiv 4 \pmod{6} \\ x + 2 \equiv 4 \pmod{7}. \end{cases}$$

5.
 - a) Find two irreducible polynomials of degree three in $\mathbb{Z}_3[x]$. How many monic irreducible polynomials are there in $\mathbb{Z}_3[x]$?
 - b) Use one of the polynomials in **a**) to construct a field with 27 elements and compute $[x^2 + 2x] + [x + 1]$, $[x^2 + x] \cdot [x^2 + 2]$ and $[x^2]^{-1}$ in this field.
6. Look at the linear code C over \mathbb{Z}_{11} with control matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{pmatrix}.$$

- a) How many words does C contain?
- b) What is the separation of C ?
- c) For each of the words $w_1 = (2\ 3\ 0\ 0\ 0\ 0\ 0\ 2\ 4)$, $w_2 = (1\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 1)$ and $w_3 = (1\ 0\ 0\ 1\ 4\ 0\ 1\ 0\ 4\ 4)$ decide if it is a code word or not. If not decide if it can be corrected. Also correct those words that can be corrected.